The coevolution of altruism and punishment: role of the selfish punisher

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Altruistic punishment

Punishment to selfish could promote the evolution of cooperation. But ....

This is a big problem!!

punisher

incur a cost to punish a selfish player

Selfish

cooperate each other

incur a cost to punish a selfish player

Altruist

Altruist-punisher (AP)

AP can make up for a cost of punishment

benefit

a fine

Selfish
The lattice promotes the evolution of cooperation, but...

Iwasa, Nakamaru and Levin (1998)

The lattice structured population promotes the evolution of spiteful behavior in a lattice.

SPITE

\[ \text{damage} \]

incur a cost to reduce others’ fitness

Spiteful behavior

\[ \text{from the viewpoint of the payoff} \]

Punishment


The score-dependent viability model also promotes the evolution of spiteful behavior in a lattice.

It is suggested that the lattice and the viability model enable punishers to evolve.
Why does lattice promote the evolution of spite, especially in the Viability model?

The viability model = “score = survivorship”

Spite has more chance to get an empty site in the neighborhood of spite.

If spite succeeds in colonizing a new open site,

It dies

The neighbor’s score reduces

Its survivorship is low

spite
damage

spite

It dies

spite can increase in number
There are four possible strategies:

<table>
<thead>
<tr>
<th></th>
<th>punisher</th>
<th>nonpunisher</th>
</tr>
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<tbody>
<tr>
<td><strong>Altruist</strong></td>
<td>A P</td>
<td>A N</td>
</tr>
<tr>
<td></td>
<td>altruist-punisher</td>
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paradoxical strategy!

Can Selfish Punisher suppress an increase of Pure Selfish and then promote the evolution of cooperation?
## Payoff Matrix

$$q = c = 1$$  
$$b, c, p, q > 0$$

<table>
<thead>
<tr>
<th></th>
<th>AP</th>
<th>AN</th>
<th>SP</th>
<th>SN</th>
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<tbody>
<tr>
<td>AP</td>
<td>$b-c$</td>
<td>$b-c$</td>
<td>$-c-q$</td>
<td>$-c-q$</td>
</tr>
<tr>
<td>AN</td>
<td>$b-c$</td>
<td>$b-c$</td>
<td>$-c$</td>
<td>$-c$</td>
</tr>
<tr>
<td>SP</td>
<td>$b-p$</td>
<td>$b$</td>
<td>$-q-p$</td>
<td>$-q$</td>
</tr>
<tr>
<td>SN</td>
<td>$b-p$</td>
<td>$b$</td>
<td>$-p$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

- **A benefit from cooperation**
  - An opponent: $b$
  - AP or AN: $-c$
  - SN or SP: $-p$
  - AP or SP: $-q$

- **A cost of cooperation**
  - An opponent: $-c$

- **A fine of punishment**
  - An opponent: $-p$

- **A cost of punishing**
  - An opponent: $-q$
Each interacts with four players chosen randomly.

The score of this “AP”
\[ = 2E[AP/AP] + E[AP/AN] + E[AP/SP] \]

Each only interacts with four nearest neighbors.

The score of this “AP”

Analyzed by the computer simulation and the ordinary differential equation.
Nakamaru & Iwasa (2005) -Altruist punisher (AP) vs. Pure selfish (SN)-

<table>
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<tr>
<th>Updating rule</th>
<th>population structure</th>
<th>complete mixing population</th>
<th>lattice-structured population</th>
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<td>score-dependent fertility model</td>
<td>mixing population</td>
<td>[ a \text{ always wins} ]</td>
<td>[ \text{SN always wins} ]</td>
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The lattice promotes the evolution of Altruist-Punisher.

Punishment affects the evolutionary dynamics in Viability model.

How does “Selfish Punisher” play its role in these 4 models?
the Score-dependent fertility model (the Fertility model)

“Birth rate” affects the dynamics of evolution.

It dies randomly

One of 4 neighbors who has a highest score can colonize the empty site with highest probability.

score $\rightarrow$ colonization probability
The result of the Fertility model in the complete mixing population

Pure Selfish (SN) wins against others

Bistability

Selfish-Punisher never wins against others.
the evolutionary dynamics in a two-dimensional lattice model of the Fertility model

\[ b = 5, \ p = 1 \]
initial \( AP = 0.3, \) initial \( AN = 0.3, \) initial \( SP = 0.3, \) initial \( SN = 0.1 \)

\[ time = 0 \]

\[ time = 200 \]
The lattice of the Fertility model

AN+AP = 0.5
SN+SP = 0.5
50x50 lattice

SN & AP only

Altruist wins

SN wins

high AN
the Score-dependent viability model (the Viability model)

“Survivorship” affects the dynamics of evolution.

One who obtains a high score dies with low probability.

One of 4 neighbors colonizes the empty site randomly.

score $\rightarrow$ survivorship
The result of the Viability model in the complete mixing model

The diagram illustrates the outcomes based on different parameters:

- **Altruist (AP)** wins against others when $b = 0.03, AN = 0.9, SP = 0.03, SN = 0.04$.

- **Selfish-Punisher (SP)** wins against others when $b = 3p - 1$. This occurs in the bistable region where $p > 4q$. In this region, **SN** decreases, **Altruist (AP)** increases, and **SP** wins against **SN**.

- **Selfish (SN)** wins against others when $b = 4p - 5$. In the bistable region, **SN** decreases, and **SP** wins against **SN**.

The **Altruist + Selfish (AN)** wins against others when $b = p - 8$. The key to understanding these outcomes is $p = 4q$, which is highlighted on the diagram.

The diagram is a visual representation of the conditions under which each strategy wins, based on the parameters $b$, $p$, and $q$.
The lattice of the viability model

AN+AP = 0.5
SN+SP = 0.5
50x50 lattice

SN & AP only

SN wins

Altruist wins

high SP

high AN
Summary

The role of Selfish Punishment (SP), a paradoxical strategy

The complete mixing population

The Fertility model
- Basically SP has no effect on the evolutionary dynamics

The Viability model
- SP encourages the evolution of Altruist-Punisher (AP)
  - SP itself evolves

The lattice population of both models

SP encourages the evolution of AP
AN discourages the evolution of AP
Another interpretation as the decision-making process models

The score-dependent fertility model

A focal player decides to imitate a behavior of a neighbor with a high score (= attractive or socially successful).

- the behavior of a cooperator tends to spread

The score-dependent viability model

A focal player with a low score makes a decision to quite his/her behavior and imitate a behavior of a neighbor chosen randomly

- the spiteful behavior tends to spread

commonly used

Take it into account!